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# Heavy quarkonium states with the holographic potential

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Abstract: The quarkonium states in a quark-gluon plasma is examined with the heavy quark potential implied by the holographic principle. Both the vanila AdS-Schwarzschild metric and the one with an infrared cutoff are considered. The dissociation temperature is calculated by solving the Schrödinger equation of the potential model. In the case of the AdS-Schwarzschild metric with a IR cutoff, the ratios of the dissociation temperatures for $J/\psi$ and $\Upsilon$ with the U-ansatz of the potential to the deconfinement temperature are found to agree with the lattice results within a factor of two.

Keywords: Lattice QCD, Phenomenological Models, AdS-CFT Correspondence.

## Contents



## 1. Introduction

AdS/CFT duality opens a new avenue towards a qualitative or even semi-quantitative understanding of the nonperturbative aspect of a quantum field theory [1, 2]. Because of the isomorphism between the conformal group in four dimensions and the isometry group of $AdS_5$ space, it is conjectured that a string theory formulated in $AdS_5 \times S_5$ is dual to the $N = 4$ supersymmetric Yang-Mills theory (SUSY YM) on the boundary. The latter is believed to be conformal at quantum level. The global SU(4) symmetry of its $R$-charges corresponds to the isometry of $S_5$. In particular, the low energy limit of the classical string theory, the supergravity in $AdS_5 \times S_5$ corresponds to the supersymmetric Yang-Mills theory at large $N_c$ and large 't Hooft coupling.

$$\lambda \equiv N_c g_{\rm YM}^2 \tag{1.1}$$

Various field theoretic correlation functions can be extracted from the metric fluctuations of the gravity dual and the expectation value of a Wilson loop operator is related to the minimum area the loop spans in the $AdS_5$ bulk.

Notable success has been made in the application of the AdS/CFT duality to the physics of quark-gluon plasma (QGP) created by RHIC, even though the underlying dynamics of QCD is very different from that of a supersymmetric Yang-Mills theory [3–5]. Among them are the viscosity-entropy ratio [3], the jet quenching parameter [4] which are closer to the observed values than the perturbative results. While it is premature to conclude that every aspect of RHIC physics can be explained in terms of SUSY YM, the AdS/CFT duality provides unprecedented references since this is the only case where the strong coupling properties of a quantum field theory can be calculated reliably.

The heavy quarkonium dissociation is an important signal of the formation of QGP in RHIC. The subject has been explored extensively on a lattice [6, 7]. In the deconfinement phase of QCD, the range of the binding potential between a quark and an antiquark is limited by the screening length in a hot and dense medium, which decreases with an increasing temperature. Beyond the dissociate temperature, $T_d$, the range of the potential



JHEP01(2008)029

is too short to hold a bound state and the heavy quarkonium will melt. The lattice simulation of the quark-antiquark potential and the spectral density of hadronic correlators yield consistent picture of the quarkonium dissociation and the numerical values $T_d$. It is the object of this paper to calculate $T_d$ using the heavy quark potential of $N = 4$ SUSY YM extracted from its gravity dual [8 − 10]. Although the potential model applies only in the non-relativistic limit which is not the case when the 't Hooft coupling, $\lambda$, becomes too strong, it can be justified within the lower side of the range of $\lambda$ used in the literature to compare AdS/CFT with the RHIC phenomenology, i.e.

$$5.5 < \lambda < 6\pi. \tag{1.2}$$

The upper edge is obtained by substituting into (1.1) $N_c = 3$ and the QCD value of $g_{\rm YM}$ at RHIC energy scale ($g^2_{\rm YM}/(4\pi) \simeq 1/2$) and the lower edge is based on a comparison between the heavy quark potential from lattice simulation with that from AdS/CFT [11].

We model the quarkonium, $J/\psi$ or $\Upsilon$, as a non-relativistic bound state of a quark and an antiquark. The wave function for their relative motion satisfies the Schrödinger equation

$$-\frac{1}{2m}\nabla^2\psi + V_{\rm eff.}(r)\psi = -E\psi \tag{1.3}$$

where $m = M/2$ is the reduced mass with $M$ the mass of the heavy quark and $E(\geq 0)$ is the binding energy of the bound state. Because of the screening of QGP, the effective potential energy has a finite range and is temperature dependent. The dissociation temperature of a particular state is the temperature at which its energy, $-E$, is elevated to zero.

In the next section we shall calculate the dissociation temperature semi-analytically with the heavy quark potential extracted from the vanila AdS-Schwarzschild metric. The case with an infrared cutoff will be examined in the section III and the section IV will conclude the paper.

## 2. The holographic potential model from the vanila AdS/CFT

The free energy of a static pair of $q\bar{q}$ separated by a distance $r$ at temperature $T$ is given

$$e^{-\frac{1}{T}F(r,T)} = \frac{{\rm tr} < W^\dagger(L_+)W(L_-) >}{{\rm tr}< W^\dagger(L_+) >< W(L_-) >} \tag{2.1}$$

where $L_\pm$ stands for the Wilson line running in Euclidean time direction at spatial coordinates $(0, 0, \pm\frac{1}{2}r)$ and is closed by the periodicity. We have

$$W(L_\pm) \equiv Pe^{-i\int_0^{\frac{1}{T}} dtA_0(t,0,0,\pm\frac{1}{2}r)} \tag{2.2}$$

with $A_0$ the temporal component of the gauge potential subject to the periodic boundary condition, $A_0(t + \frac{1}{T}, \vec{r}) = A_0(t, \vec{r})$. The trace in (2.2) is over the color indexes and $< \ldots >$ denotes the thermal average. The symbol $P$ enforces the path ordering. The corresponding internal energy reads

$$U(r, T) == -T^2\frac{d}{dT}\left(\frac{F(r, T)}{T}\right) \tag{2.3}$$



JHEP01(2008)029

In the de-confined phase, the numerator of (2.1) factorizes at large separation, i.e.

$$\lim_{r\to\infty} < W^\dagger(L_+)W(L_-) >=< W^\dagger(L_+) >< W(L_-) > \tag{2.4}$$

and therefore $\lim_{r\to\infty} F(r,T) = \lim_{r\to\infty} U(r,T) = 0$. The thermal average of a single Wilson line, $< W(L) >$, is independent of its spatial coordinates.

Two ansatz of the potential model have been explored in the literature [7]: the $F$-ansatz which identifies $V_{\rm eff.}$ of (1.3) with $F(r,T)$ and the $U$-ansatz which identifies $V_{\rm eff.}$ with $U(r,T)$. The lattice QCD simulation reveals that the $U$ ansatz produces a deeper potential well and thereby higher $T_d$ because the entropy contribution is subtracted. This remains the case with holographic potential as we shall see.

According to the holographic principle, the thermal average of a Wilson loop operator

$$W(C) = Pe^{-i\oint_C dx^\mu A_\mu(x)} \tag{2.5}$$

in 4D $N = 4$ SUSY YM at large $N_c$ and large 't Hooft coupling corresponds to the minimum area $S_{\rm min.}[C]$ of the string world sheet in the 5D AdS-Schwarzschild metric with a Euclidean signature,[1]

$$ds^2 = \pi^2T^2y^2(fdt^2 + d\vec{x}^2) + \frac{1}{y^2f}dy^2, \tag{2.6}$$

bounded by the loop $C$ at the boundary, $y \to \infty$, where $f = 1 - \frac{1}{y^4}$. Specifically, we have

$$\mathrm{tr} < W(C) >= e^{-\sqrt{\lambda}S_{\rm min}[C]}. \tag{2.7}$$

For the numerator of (2.1), $C$ consists of two parallel temporal lines $(t, 0, 0, \pm\frac{r}{2})$ and the string world sheet can be parameterized by $t$ and $y$ with the ansatz $x_1 = x_2 = 0$ and $x_3$ a function of $y$. The induced world sheet metric reads

$$ds^2 = \pi^2T^2y^2fdt^2 + \Big[\pi^2T^2y^2\Big(\frac{dx_3}{dy}\Big)^2 + \frac{1}{\pi^2T^2y^2f}\Big]dy^2. \tag{2.8}$$

Minimizing the world sheet area (the Nambu-Goto action)

$$S[C] = (\pi T)\int_0^{\frac{1}{T}} dt \int_0^\infty dy\sqrt{1 + \pi^4T^4y^4f\Big(\frac{dx_3}{dy}\Big)^2} \tag{2.9}$$

generates two types of solutions [8−10]. One corresponds to a single world-sheet with a nontrivial profile $x_3(y)$,

$$x_3 = \pm\pi Tq\int_{y_c}^y \frac{dy'}{\sqrt{(y'^4-1)(y'^4-y_c^4)}} \tag{2.10}$$

where $q$ is a constant of integration determined by the boundary condition

$$r = \frac{2q}{\pi T}\int_{y_c}^\infty \frac{dy}{\sqrt{(y^4-1)(y^4-y_c^4)}} \tag{2.11}$$

[1]The Wilson loops considered throughout this paper are assumed to have a trivial projection (a point) onto $S_5$ sector. Therefore only the $AdS_5$ metric of the garvity dual is shown. An average over the $S_5$ sector is proposed recently in [12].



JHEP01(2008)029

with $y_c^4 = 1 + q^2$. The corresponding value of $\sqrt{\lambda}S[C]$ is denoted by $I_1$. The other solution consists of two parallel world sheets with $x_3 = \pm\frac{r}{2}$ extending to the black hole horizon and the corresponding value of $\sqrt{\lambda}S[C]$ is denoted by $I_2$. The latter solution corresponds to two non-interacting static quarks in the medium and is equal to the denominator of (2.1)

The free energy we are interested in reads

$$F(r,T) = T\min(I,0) \tag{2.12}$$

where

$$I \equiv I_1 - I_2 = \sqrt{\lambda}\bigg[\int_{y_c}^{\infty} dy \left(\sqrt{\frac{y^4-1}{y^4-y_c^4}} - 1\right) + 1 - y_c\bigg]. \tag{2.13}$$

Inverting eq. (2.11) to express $q$ in terms of $r$ and substituting the result to (2.13), it was found that the function $I$ consists of two branches, The upper branch is always positive and is therefore unstable. The lower branch starts from being negative for $r < r_0$ and becomes positive for $r > r_0$. Both branches joins at $r = r_c > r_0$ beyond which the nontrivial solution ceases to exist. Numerically, we have $r_0 \simeq \frac{0.7541}{\pi T}$ and $r_c \simeq \frac{0.85}{\pi T}$. Introducing a dimensionless radial coordinate,

$$\rho = \pi T r, \tag{2.14}$$

we find that

$$F(r,T) = -\frac{\alpha}{r}\phi(\rho)\theta(\rho_0 - \rho), \tag{2.15}$$

where $\alpha = \frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\sqrt{\lambda} \simeq 0.2285\sqrt{\lambda}$. The screening factor $\phi(\rho) = -\rho I/(\pi\alpha)$ and is shown in figure 1a. We have $\phi(0) = 1$ and $\phi(\rho_0) = 0$ with

$$\rho_0 = 0.7541. \tag{2.16}$$

The small $\rho$ expansion of $\phi(\rho)$ is given by

$$\phi(\rho) = 1 - \frac{\Gamma^4\left(\frac{1}{4}\right)}{4\pi^3}\rho + \frac{3\Gamma^8\left(\frac{1}{4}\right)}{640\pi^6}\rho^4 + O(\rho^8). \tag{2.17}$$

On writing the wave function $\psi(\vec{r}) = u_l(\rho)Y_{lm}(\hat{r})$, the radial Schrödinger equation for a zero energy bound state reads

$$\frac{d^2u_l}{d\rho^2} + \frac{2}{\rho}\frac{du_l}{d\rho} - \left[\frac{l(l+1)}{\rho^2} + \mathcal{V}\right]u_l = 0 \tag{2.18}$$

with $\mathcal{V} = MV_{\text{eff.}}/(\pi^2T^2)$. We have

$$\mathcal{V} = -\frac{\eta^2}{\rho_0\rho}\phi(\rho)\theta(\rho_0 - \rho) \tag{2.19}$$

for the F ansatz and

$$\mathcal{V} = -\frac{\eta^2}{\rho_0\rho}\left[\phi(\rho) - \rho\left(\frac{d\phi}{d\rho}\right)\right]\theta(\rho_0 - \rho) \tag{2.20}$$

for the U ansatz, where

$$\eta = \sqrt{\frac{\alpha\rho_0 M}{\pi T}}. \tag{2.21}$$

Note that the potential of the U-ansatz jumps to zero from below at $\rho = \rho_0$, since the derivative of $\phi(\rho)$ is nonzero there. For both ansatz, and the case with an infrared cutoff discussed in the next section, the solution to (2.18) is given by[2]

$$u_l = \text{const.}\rho^{-l-1} \tag{2.22}$$

at $\rho > \rho_0$ and by

$$u_l = \text{const.}\rho^l \tag{2.23}$$

near the origin. The threshold value of $\eta$ at the dissociation temperature, $\eta_d$, is determined by the matching condition at $\rho = \rho_0$,

$$\frac{d}{d\rho}(\rho^{l+1}u_l)\mid_{\rho=\rho_0^-}= 0. \tag{2.24}$$

It follows from (2.21) that the dissociation temperature is given by

$$T_d = \frac{\alpha\rho_0 M}{\pi\eta_d^2} = \frac{4\pi\rho_0}{\Gamma^4\left(\frac{1}{4}\right)\eta_d^2}\sqrt{\lambda}M. \tag{2.25}$$

It is interesting to observe that the extrapolation of the first two terms of (2.17) vanishes at

$$\rho = \rho_0' = \frac{4\pi^3}{\Gamma^4\left(\frac{1}{4}\right)} \simeq 0.7178, \tag{2.26}$$

which is very close to the exact zero point (2.16), and the third term of (2.17) remains small there. This suggests that the screening factor $\phi(\rho)$ can be well approximated by a linear function

$$\bar{\phi}(\rho) \simeq 1 - \frac{\rho}{\bar{\rho}_0} \tag{2.27}$$

with $\bar{\rho}_0 = \frac{1}{2}(\rho_0 + \rho_0') \simeq 0.7359$, as is evident from the exact profile of $\phi(\rho)$ shown in figure 1a. The effective potential $V_{\rm eff.}$ is then approximated by a truncated Coulomb potential. We have

$$\mathcal{V} = -\frac{\eta^2}{\rho_0\rho}\left(1 - \frac{\rho}{\rho_0}\right)\theta(\rho_0 - \rho) \tag{2.28}$$

for the F-ansatz and

$$\mathcal{V} = -\frac{\eta^2}{\rho_0\rho}\theta(\rho_0 - \rho) \tag{2.29}$$

for the U-ansatz, where the over bar of $\rho_0$ has been suppressed.

The radial wave function of the F-ansatz under the truncated Coulomb approximation can be expressed in terms of the confluent hypergeometric function of the 1st kind for $\rho < \rho_0$, i.e.

$$u_l = \rho^l {}_1F_1(l + 1 - \frac{\eta}{2}; 2l + 2; 2\eta\frac{\rho}{\rho_0}). \tag{2.30}$$

[2]The exponential decay factor that ensures the normalizability of a bound state wave function approaches to one in the limit of zero binding energy.

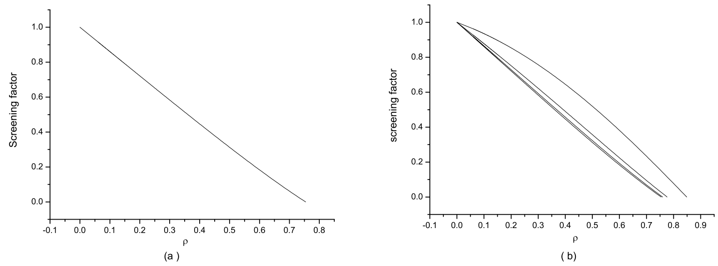


**Figure 1:** (a)The exact screening factor $\phi(\rho)$ profile extracted from the metric (2.6). (b) The screening factors $\chi(\rho, T)$'s extracted from the metric (3.5) at different ratios of $T/T_c = 1, 2, 3, \infty$ from top to bottom.

The matching condition (2.24) yields the secular equation for $\eta$,

$$2l + 1 - \eta + \eta \left(1 - \frac{\eta}{2l+2}\right) \frac{{}_1F_1(l + 2 - \frac{\eta}{2}; 2l + 3; 2\eta)}{{}_1F_1(l + 1 - \frac{\eta}{2}; 2l + 2; 2\eta)} = 0. \tag{2.31}$$

As $\eta$ is reduced from above, we expect the bound states of the same $l$ to melt successively. Therefore the first positive root corresponds to the minimum binding strength for a bound state of angular momentum $l$ and the 2nd one to the threshold of the first radial excitation. Knowing the values of these $\eta$'s, the disassociation temperature can be computed from the formula (2.25). For example, the threshold $\eta$ of the $1S$ state, $\eta_{1S} \simeq 1.76$, which implies that

$$T_d \simeq 0.0173\sqrt{\lambda} M. \tag{2.32}$$

In case of the $U$-ansatz under the same approximation, we find that

$$u_l = \frac{1}{\sqrt{\rho}} J_{2l+1}\left(2\eta\sqrt{\frac{\rho}{\rho_0}}\right) \tag{2.33}$$

for $\rho < \rho_0$ with $J_\nu(x)$ the Bessel function. The secular equation for $\eta$ reads

$$2l + 1 - \eta \frac{J_{2l+2}(2\eta)}{J_{2l+1}(2\eta)} = 0. \tag{2.34}$$

We have $\eta_{1S} = 1.20$ and

$$T_d \simeq 0.0370\sqrt{\lambda} M. \tag{2.35}$$

Numerical results for the dissociation temperature of quarkonium are tabulated in table 1, where we have used the mass values $M = 1.65\text{GeV}$, $4.85\,\text{GeV}$ for $c$ and $b$ quarks [13]. The errors caused by the truncated Coulomb approximation are within 4 percent, as is shown by the numerical solution to the Schrödinger equation of the exact potential.

To assess the validity of the potential model employed, we consider a classical two body problem with the truncated Coulomb potential (2.12). The Lagrangian of the system reads

$$L = \frac{1}{2} M(\dot{\vec{r}}_1^{\,2} + \dot{\vec{r}}_2^{\,2}) - V_{\text{eff.}}(|\vec{r}_1 - \vec{r}_2|) \tag{2.36}$$



JHEP01(2008)029

| ansatz | $J/\psi(1S)$ | $J/\psi(2S)$ | $J/\psi(1P)$ | $\Upsilon(1S)$ | $\Upsilon(2S)$ | $\Upsilon(1P)$ |
|---|---|---|---|---|---|---|
| $F$ | 67-124 | 15-28 | 13-25 | 197-364 | 44-81 | 40-73 |
| $U$ | 143-265 | 27-50 | 31-58 | 421-780 | 80-148 | 92-171 |

**Table 1:** $T_d$ in MeV's under the truncated Coulomb approximation. The lower value of each entry corresponds to $\lambda = 5.5$ and the upper one to $\lambda = 6\pi$.

Introducing the center of mass coordinates and the relative coordinates via $\vec{r}_1 = \vec{R} + \vec{r}/2$ and $\vec{r}_2 = \vec{R} - \vec{r}/2$, and assuming a static pair, $\dot{\vec{R}} = 0$, we have

$$L = \frac{1}{4} M \dot{\vec{r}}^2 - V_{\text{eff.}}(r) \tag{2.37}$$

which becomes a particle of the reduced mass in an central potential $V_{\text{eff.}}(r)$. For a circular orbit, the force balance at the border of the potential well, $r = d \equiv \rho_0/(\pi T)$ is given by

$$\frac{M v_r^2}{2d} = \frac{\alpha}{d^2}. \tag{2.38}$$

which yields

$$v^2 = \frac{v_r^2}{4} = \frac{\alpha^2}{2\eta^2} \tag{2.39}$$

with $\vec{v} = \dot{\vec{r}}_1 = -\dot{\vec{r}}_2$ and $\vec{v}_r = \dot{\vec{r}}$. At the dissociation temperature of U-ansatz, we find that $0.099 < v^2 < 0.340$ for $\lambda$ in the range of (1.2). Therefore the approximation is less reliable at the upper limit of the range (1.2). On the other hand, the lower values of $\alpha$ was advocated in [11] based on a comparison between the potentials from lattice simulation and that from the AdS/CFT and may serve our purpose better. The NR approximation works better for the dissociation temperatures of excitations because of the higher $\eta_d$ values.

## 3. The holographic potential model with an infrared cutoff

Because of the conformal invariance at quantum level, there is no color confinement in $N = 4$ SUSY YM even at zero temperature. In order to simulate the confined phase of QCD at low temperature, an infrared cutoff has to be introduced that suppress the contribution of the AdS horizon. Two scenarios explored in the literature are the hard-wall model and the soft-wall model [14, 15]. The gravity dual of the de-confinement transition is modeled as the Hawking-Page transition from a metric without a black hole at $T < T_c$ to that with a black hole at $T > T_c$. The gravity dual of the free energy with a hard wall is the Einstein-Hilbert action with a cosmological constant given by

$$F = -\frac{T}{16\pi G_5} \int d^4x \int_0^{z_0} dz \sqrt{g}(R - 12), \tag{3.1}$$

subject to an appropriate UV regularization, where $R$ is the curvature scalar and $G_5$ is the gravitational constant in 5D. In the hadronic phase, the metric underlying $g$ and $R$ is that of the standard $AdS_5$

$$ds^2 = \frac{1}{z^2}(dt^2 + d\vec{x}^2 + dz^2), \tag{3.2}$$



JHEP01(2008)029

truncated beyond $z_0$ with $z_0$ determined by the $\rho$-meson mass [14]. In the plasma phase, the underlying metric is given by

$$ds^2 = \frac{1}{z^2}(fdt^2 + d\vec{x}^2 + f^{-1}dz^2), \tag{3.3}$$

with $f = 1 - \pi^4 T^4 z^4$, which is identical to (2.6) upon the transformation $y = 1/(\pi T z)$. The domain of the $z$-integration is $0 < z < 1/(\pi T)$ which corresponds to $1 < y < \infty$. Notice that $z_0 > 1/(\pi T)$ above the transition temperature $T_c$, which was found to be $T_c \simeq 0.1574 m_\rho$ [16]. Therefore heavy quark potential and the meson dissociation temperatures calculated with the hard-wall model are identical to what calculated in the last section with the vanila AdS-Schwarzschild metric.

In case of the simplest soft-wall model ([15]), a dilaton is introduced that modefies eq. (3.1) to

$$F = -T\frac{1}{16\pi G_5}\int d^4x \int_{\rho_0}^{\infty} dr e^{-\frac{c}{\rho^2}}\sqrt{g}(R-12), \tag{3.4}$$

where $c$ is determined by the $\rho$-mass and the transition temperature is predicted as $T_c \simeq 0.2459 m_\rho$ [16]. The string frame metric underlying $g$ and $R$ remains given by (3.2) in the hadronic phase and by (3.3) in the plasma phase. Although, the infrared cutoff is partially carried over to the plasma phase, the heavy-quark potential and the dissociation temperatures thus obtained remains intact since the minimum area dual to a Wilson loop has to be defined with respect to the string frame metric.[3]

A variant of the soft-wall scenario proposed in ref. [17, 18], however, admits a string frame metric that is different from (3.3) by a conformal factor, i. e.

$$ds^2 = \frac{e^{bz^2}}{z^2}(fdt^2 + d\vec{x}^2 + f^{-1}dz^2), \tag{3.5}$$

The value of $b = 0.184\mathrm{GeV}^2$ was obtained by fitting the lattice simulated transition temperature $T_c = 186\mathrm{MeV}$ [18]. Following the steps from (2.6) to (2.13), we find that

$$F(r,T) = -\frac{\alpha}{r}\chi(\rho,T) \tag{3.6}$$

where the screening factor $\chi(\rho,T)$ is defined parametrically by

$$\chi = -\frac{\rho\sqrt{\lambda}}{\pi\alpha}\left[\int_{y_c}^{\infty} dy e^{\frac{\beta}{y^2}}\left(\sqrt{\frac{y^4-1}{y^4-1-q^2 e^{-\frac{2\beta}{y^2}}}}-1\right) - \int_1^{y_c} dy e^{\frac{\beta}{y^2}}\right]. \tag{3.7}$$

and

$$\rho = \frac{2q}{2T}\int_{y_c}^{\infty}\frac{dy e^{-\frac{\beta}{y^2}}}{\sqrt{(y^4-1)(y^4-1-q^2 e^{-\frac{2\beta}{y^2}})}} \tag{3.8}$$

with

$$q^2 = (y_c^4 - 1)e^{\frac{2\beta}{y_c^2}} \tag{3.9}$$

[3] We are obliged to James T. Liu for pointing it out to us

| ansatz | $J/\psi$ | $\Upsilon$ |
|---|---|---|
| $F$ | NA | 235-385 |
| $U$ | 219-322 | 459-780 |

**Table 2:** $T_d$ in MeV's for the $1S$ state with the deformed metric. "NA" means that there is no bound states above $T_c$ and the entry for the $\Upsilon$ with $U$ ansatz and $\alpha = 6\pi$ is taken from the table I, since no significant increment is observed.

and $\beta = \frac{b}{\pi^2 T^2} = 0.539\frac{T_c^2}{T^2}$. We have $\chi(\rho, \infty) = \phi(\rho)$. The small $\rho$ behavior of $\chi(\rho, T)$ reads

$$\chi(\rho, T) = 1 - \frac{\Gamma^4\left(\frac{1}{4}\right)}{4\pi^3} a\rho + O(\rho^2). \tag{3.10}$$

where

$$a = e^\beta - 2\beta \int_0^1 dx e^{\beta x^2}. \tag{3.11}$$

The numerical results of $\chi(\rho, T)$ for several values of the ratio $T/T_c$ are shown in figure 1b. As is seen, the screening becomes weaker in the neighborhood of $T_c$, similar to lattice QCD result [7]. The function $\chi(\rho, T)$ vanishes at $\rho_0 \simeq 0.8485$ at $T = T_c$ while the extrapolation of the first two terms of (3.10) vanishes at $\rho_0' \simeq 1.764$. The truncated Coulomb approximation deteriorates in the vicinity of $T_c$. The screening length defined by $\rho_0$ decreases quickly to its conformal limit (2.16) as $T$ increases beyond $T_c$. So does the value of $\rho_0'$. The longer screening length near $T_c$ is the consequence of the positive $b$ value in the string frame metric (3.5). The screening effect from the second term of (3.10) also becomes weaker since the factor $a$ is a decreasing function of $b$ and passe one at $b = 0$. In case of a negative $b$, the numerical result of $\rho_0$ near $T_c$ is found to be smaller than the limiting value (3.10) and its temperature dependence is reversed.

To determine the dissociation temperature in this case, we have to solve the Schrödinger equation (2.18) numerically with the numerically calculated heavy quark potential for both ansatz. We have

$$\mathcal{V} = -\frac{\eta^2}{\rho_0 \rho}\chi(\rho, T) \tag{3.12}$$

for the F ansatz and

$$\mathcal{V} = -\frac{\eta^2}{\rho_0 \rho}\left[\chi(\rho, T) - \rho\left(\frac{\partial \chi}{\partial \rho}\right)_T - T\left(\frac{\partial \chi}{\partial T}\right)_\rho\right] \tag{3.13}$$

for the U ansatz. The solution for $\rho < \rho_0$ can be obtained by standard Runge-Kutta method and the threshold value of $\eta = \eta_d$ follows from the matching condition (2.24). Notice that the the infrared cutoff renders both $\rho_0$ and $\eta_d$ nontrivial functions of temperature and eq. (2.25) becomes an *implicit* equation of $T_d$. Nor does $T_d$ scales simply with $\sqrt{\lambda}$ and $M$ according to (2.32) and (2.35).

The modified dissociation temperatures are tabulated in the table 2, which show an significant increment in the vicinity of $T_c$. The comparison between the ratios $T_d/T_c$ we calculated here with that obtained from the lattice QCD is shown in table 3.

| ansatz | $J/\psi$(holographic) | $J/\psi$(lattice) | $\Upsilon$(holographic) | $\Upsilon$(lattice) |
|---|---|---|---|---|
| $F$ | NA | 1.1 | 1.3-2.1 | 2.3 |
| $U$ | 1.2-1.7 | 2.0 | 2.5-4.2 | 4.5 |

**Table 3:** The ratio $T_d/T_c$ for the 1S state from the holographic potential and that from the lattice QCD

## 4. Concluding remarks

In summary, we have calculated the dissociation temperatures of heavy quarkonia using the static potential implied by the holographic principle with both the vanila AdS-Schwarzschild metric and the one with an infrared cutoff. While estimations of $T_d$ have been made in the literature based on various holographic models [19, 20], a determination of $T_d$ from the Schrödinger equation within the same framework remains lacking. Our work is to fill this gap. The authors of [19] gave an order of magnitude estimate of the dissociation temperature relying on the screening length only. The author of [20] generalized the spectral analysis of the light mesons to heavy mesons. Their criterion for the dissociation, however, appears slightly ad hoc and is again independent of the coupling. Both the screening length and the coupling strength ought to affect the heavy quaronium binding. Carrying out the analysis of the potential model inspired by the holographic principle to the same extent of that of QCD will address both contributions, especially the consistency of the range (1.2)of the coupling constant extracted from the jet quenching with the heavy quarkonium physics. Also a detailed bound state calculation enables us to assess the validity of the nonrelativistic approximation behind the potential model. On comparing our results with that from the lattice simulation [7], we found that our ratios $T_d/T_c$ extracted from the modified AdS-Schwarzschild metric (3.5) are lower than the lattice ones within a factor of two. That the increment in $T_d/T_c$ from the F-ansatz to the U-ansatz is about a factor of two is similar to what reported in [7]. One has to bear in mind that the lattice results reviewed in [7] were extracted from a pure SU(3) gauge theory without a matter field. On the other hand the matter field contents of $N = 4$ SUSY YM are larger than that of QCD with light quarks. It is possible that the additional screening effect of the matter field in $N = 4$ SUSY YM makes the heavy quarkonia more vulnerable and thereby lowers the dissociation temperature. This is consistent with the observation that the potential well becomes wider in the metric with the IR cutoff introduced in [17, 18] since some of degrees of freedom becomes massive.

The authors of [21] calculated the spectral function of the fluctuation of a D7 brane and deduced from which the meson melting temperature

$$T_d = \frac{2.17M}{\sqrt{\lambda}}. \tag{4.1}$$

for all bound state levels. On substituting the value of the heavy quark masses, it gives rise to 825MeV $< T_d <$ 1.53GeV for $J/\psi$ and 2.42GeV $< T_d <$ 4.85GeV for $\Upsilon$ within the range (1.2) of the 't Hooft coupling. The spectral analysis of [21] would be superior if the underlying dynamics of QGP were $N = 4$ SUSY YM. But the higher $T_d$'s may point to its difference from QCD. Also the $\lambda$ dependence of $T_d$ in eq. (4.1) is entirely at variance with ours. Our relationship $T_d \propto \sqrt{\lambda}$ for the vanila AdS-Schwarzschild metric follows from the property that the binding strength is proportional to $\sqrt{\lambda}$ but the binding range is independent of $\lambda$. A field theoretic speculation on this property of the heavy quark potential can be found in [22]. Since we are comparing two different theories, some features may be shared by both and some features may not. If the screening properties of the $N = 4$ SUSY YM can be carried over to QCD, our potential model calculation inspired by the holographic principle should provide a semi-quantitative description on the quarkonium dissociation mechanism in the realistic quark-gluon plasma.

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